Derivation of Johnson-Nyquist noise

Johnson-Nyquist or thermal noise is due to the random motion of charge carriers that is induced by thermal fluctuations. In an arbitrary 1D conductor, the equipartition theorem states that the kinetic energy due to thermal fluctuations is proportional to the temperature T via the following relation:

$$\frac{1}{2}m\overline{v^2} = \frac{1}{2}k_B T$$

Here m is the electron mass, $\overline{v^2}$ is mean square-velocity of the carriers and $k_B = 1.38 \times 10^{-23}$is the Boltzmann constant.

In a 1D conductor of length l, such as a tunnel junction, we can write the time dependent current as:

$$I(t) = \frac{q\overline{v(t)}}{l}$$

Here q is the electric charge. The autocorrelation of $I(t)$ can then be computed as:

$$< I(t)I(t+t') > = \frac{q^2 < \overline{v(t)}\,\overline{v(t+t')} >}{l^2}$$

Drude Assumptions: 1. Electrons do not interact with each other.

2. Electron motion is only influenced by elastic scattering

We can then define the average time τ between two scattering events between an electron and a scattering center. The scattering probability at any time is then given by:

$$\wp \sim e^{-t/\tau}$$

So that the autocorrelation function of the velocity is given by:

$$< \overline{v(t)}\,\overline{v(t+t')} > = \overline{v^2}e^{-t/\tau}$$

Qualitatively, what this means is that the velocity de-correlates from its previous values versus *t* as a decay exponential.

The current autocorrelation can then be written as:

$$< I(t)I(t+t') > = \frac{q^2\overline{v^2}e^{-t/\tau}}{l^2}$$

And following the equipartition theorem:

$$< I(t)I(t+t') > = \frac{q^2 k_B T}{ml^2}e^{-t/\tau}$$

For a continuous flux of *N* electrons,

$$< I(t)I(t+t') > = \frac{Nq^2 k_B T}{ml^2}e^{-t/\tau} = \frac{(nAl)q^2 k_B T}{ml^2}e^{-t/\tau}$$

Here we have replaced *N* by *nAl* where *n* is the density of electrons per unit volume, *A* is the cross-sectional area through which the electron propagate and *l* is the length of the conductor. Hence,

$$< I(t)I(t+t') > = \frac{nAq^2 k_B T}{ml}e^{-t/\tau}$$

The current noise spectral density S_I is defined as:

$$S_I = 2\mathcal{F}\{< I(t)I(t+t') >\}$$

$\mathcal{F}\{< I(t)I(t+t') >\}$ is the Fourier transform of the current autocorrelation.

$$S_I = \frac{2nAq^2k_BT}{ml}\mathcal{F}\{e^{-t/\tau}\}$$

$$S_I = \frac{2nAq^2k_BT}{ml}\frac{2\tau}{1+\omega^2\tau^2}$$

In a typical conductor, τ is on order to 1 picosecond at most, where as ω in noise measurements, is on the order to 1GHz at most. There $\omega^2\tau^2 \sim 10^{-6}$ at most, and can therefore be neglected to get:

$$S_I = \frac{4nAq^2k_BT}{ml}\tau$$

Recall the Drude conductivity of a conductor, given by:

$$\sigma = \frac{nq^2\tau}{m}$$

We then get:

$$S_I = 4k_BT\frac{A\sigma}{l}$$

$$\frac{A\sigma}{l} = \frac{1}{R}$$

R is the resistance of the conductor/

So finally we have the current noise spectral density in terms of quantities that are simple to measure:

$$S_I = \frac{4k_BT}{R}$$

From this relation we can deduce the voltage noise spectral density S_V due to Johnson-Nyquist noise:

$$S_V = 4k_BTR$$

Other types of noise

Other types of noise that are common to conductors and semiconductors are *1/f* noise and generation-recombination noise. 1/*f* noise dominates at low frequency and varies as the inverse of the frequency. It is due to material imperfections that arise from fabrication and synthesis processes. Generation-recombination noise is common in semiconductor devices and is due to fluctuations in the number of charge carriers in the conductor. Other less common or more peculiar types of noise include (*1/f*^{2} noise, burst noise, avalanche noise).

I leave the task to deriving shot noise to you, as part of the report that is submitted at the end of the project.